Instructions for All Parts (Please specify)

1. The full mark for this examination is 100.

2. Calculation can be carried out in a calculator or electronic dictionary. Other computational tools, such as mobile phone, are not allowed.

1. (30 marks)
   Answer the following questions:

(a) It is known that $600 invested for two years will earn $264 in interest. Find the accumulated value of $2000 invested at the same rate of compound interest for three years. (10 marks)

(b) The amount of interest earned on A for one year is $336, while the equivalent amount of discount is $300. Find A. (10 marks)

(c) Fund A accumulates at a simple interests rate of 10%. Fund B accumulates at a simple discount rate of 5%. Find the point in time at which the forces of interest on the two funds are equal. (10 marks)

(a) $600 (1+i)^2 = 600 + 264$

$2000 (1+i)^3 = 3456$

(b) $\begin{cases} A_i = 336 \\ A_d = 300 \end{cases}$

$A_i - A_d = A(i-d) = 36$

$A_i \cdot A_d = A^2 \cdot i \cdot d = 336 \cdot 300$

$\frac{36}{A} = \frac{336 \cdot 300}{A^2}$

$A = 2800$

(c) $A_A(t) = 1 + 0.1t$

$A_B(t) = (1-0.05t)^{-1}$

$\frac{A_A(t)}{A_B(t)} = \frac{A_A'(t)}{A_B'(t)} = \frac{0.1}{1+0.1t} = \frac{0.1}{1-1.05t}$

$t = 5$
2. (20 marks)
You are given that \( A(t) \) is a second degree polynomial for \( 0 \leq t \leq 2 \) and that \( A(0) = 100, A(1) = 110, \) and \( A(2) = 136. \)

(a) Determine \( i_2. \)  
(b) Determine the equivalent effective rate of discount between \( t = .5 \) and \( t = 1.5. \)  
(c) Determine \( \delta_{1.2}. \)
(c) Find the present value of 1 to be paid at time \( t = 1.25 \) evaluated at time \( t = .75. \)

\[
A(t) = at^2 + bt + c
\]

\[
\begin{align*}
A(0) &= 100 \\
A(1) &= 110 \quad \Rightarrow \quad a = 8, \quad b = 2, \quad c = 100 \\
A(2) &= 136
\end{align*}
\]

\[
\therefore A(t) = 8t^2 + 2t + 100
\]

(a). \( i_2 = \frac{A(2) - A(1)}{A(1)} = \frac{136 - 110}{110} = 23.6\% \)

(b). \( \delta = \frac{A(1.5) - A(0.5)}{A(1.5)} = \frac{121 - 103}{121} = 14.88\% \)

(c). \( S_t = \frac{A(t)}{A(t)} = \frac{16t + 2}{8t^2 + 2t + 100} \)

\[
S_{1.2} = 21.2/113.92 = 18.6\%
\]

(d). \( i = \frac{A(1.25) - A(0.75)}{A(0.75)} = \frac{115 - 106}{106} = 0.0849. \)

\[
\frac{i}{i + i} = 0.9217
\]
3. (10 marks)

It is known that \( 1 + \frac{r}{n} = (1+i)^\frac{1}{n} \) Find \( n \).

\[
1 + \frac{\frac{3}{4}}{1} = (1+i)^\frac{1}{n} \quad \text{and} \quad 1 + \frac{\frac{2}{5}}{1} = (1+i)^\frac{1}{n}
\]

\[
(1+i)^\frac{1}{n} = \frac{\frac{3}{4}}{1} = \frac{\frac{2}{5}}{1} = (1+i)^\frac{1}{n}
\]

\[
\therefore n = 20
\]

4. (10 marks)

It is known that \( \frac{a_{67}}{a_{11}} = \frac{a_{9} + a_{9}}{a_{9} + a_{9}} \) Find \( x, y, \) and \( z \).

\[
\begin{array}{cccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\
A_{11} & & & & & & & & & & & \\
A_{11} & & & & & & & & & & &
\end{array}
\]

\[
A_{11} = 0.01 + 0.01 \quad \text{the current value at } t=2 \\
A_{11} + \hat{A}_{11} \quad \text{and} \quad A_{11} + \hat{A}_{11} \quad \text{respectively.}
\]

\[
X = 4, \quad Y = 6, \quad Z = 5
\]

5. (10 marks)

Money accumulates at a varying force of interest

\[
\delta = 0.05 + 0.01t \quad \text{for } 0 \leq t \leq 4
\]

Find the present value at time \( t = 0 \) of two payments of 100 each to be paid at times \( t = 2 \) and \( t = 4 \).

\[
\begin{array}{cccc}
0 & 2 & 4 \\
\hline
100 & 100 \\
\end{array}
\]

\[
\delta(t) = e^{-\int_{0}^{t} 0.05 + 0.01k \, dk} = e^{-0.05t^2 - 0.005k^2}_{t=0} = e^{-0.05t - 0.005t^2}
\]

the present value at time \( t = 0 \) is

\[
100 \delta(t) = 100 + 100 \delta(t) = 88.69204 + 71.57837 = 160.27141
\]
6. (10 marks)
You can receive one of the following two payments streams:

1. $100 at time 0, $200 at time n, and $300 at time 2n.

2. $600 at time 10.

At an annual effective interest rate of \( i \), the present values of two streams are equal. Given \( e^n = 0.75911 \), determine \( i \).

\[
\begin{align*}
100 + 200v^n + 300v^{2n} &= 600v^{10} \\
v^{10} &= (1+i^2)^{-10} = 0.7082 \\
\Rightarrow i &= (0.7082)^{10} = 0.035105
\end{align*}
\]

7. (10 marks)
A level perpetuity-immediate is to be shared by A, B, C, and D. A receives the first \( n \) payments, B the second \( n \) payments, C the third \( n \) payments, and D all payments thereafter. It is known that the ratio of the present value of C's share to A's share is 0.49. Find the ratio of the present value of B's share to D's share.

\[
\begin{align*}
\text{Suppose the amount of each payment is } R. \\
\text{then the present value:} \\
\text{for A: } R \cdot a_n \\
\text{for B: } R \cdot a_n \cdot V^n \\
\text{for C: } R \cdot a_n \cdot V^{2n} \\
\text{for D: } R \cdot \frac{1}{2} \cdot V^{3n}
\end{align*}
\]

\[
\begin{align*}
\frac{C}{A} &= \frac{R \cdot a_n \cdot V^{2n}}{R \cdot a_n} = 0.47. \Rightarrow V^n = 0.7 \\
\frac{B}{D} &= \frac{R \cdot a_n \cdot V^n}{R \cdot \frac{1}{2} V^{3n}} = \frac{V^n}{\frac{1}{2} V^{3n}} = \frac{1 - V^n}{V^n} \\
&= \frac{1 - 0.7}{0.7} \\
&= 0.612
\end{align*}
\]